

1. What is the speed of an alpha particle after it leaves the accelerator? The bias voltage applied across the quartz bottle is 5 kV for Helium.

$$\frac{1}{2}m_{\alpha}v_{\alpha}^2 = q_{\alpha}^i \Delta V_{\text{bottle}} + q_{\alpha}^{\text{II}} \Delta V_{\text{accelerate to terminal}} + q_{\alpha}^{\text{III}} \Delta V_{\text{accelerate away from terminal}}$$

$$\frac{1}{2}(2 \times 1.69 \times 10^{-27} \text{ kg} + 2 \times 1.67 \times 10^{-27} \text{ kg})v_{\alpha}^2 = (2 \times 1.6 \times 10^{-19} \text{ C} \times 5000 \text{ V}) + (1 \times 1.6 \times 10^{-19} \text{ C} \times 1.1 \text{ MV}) + (2 \times 1.6 \times 10^{-19} \text{ C} \times 1.1 \text{ MV})$$

$$3.36 \times 10^{-27} \text{ kg} v_{\alpha}^2 = 1.6 \times 10^{-15} \text{ J} + 1.76 \times 10^{-13} \text{ J} + 3.52 \times 10^{-13} \text{ J} = 5.296 \times 10^{-13} \text{ J}$$

$$\therefore v_{\alpha}^2 = \frac{5.296 \times 10^{-13} \text{ J}}{3.36 \times 10^{-27} \text{ kg}} = 1.576 \times 10^{14} \frac{\text{m}^2}{\text{s}^2} \rightarrow v_{\alpha} = 1.256 \times 10^7 \frac{\text{m}}{\text{s}}$$

2. If the radius of the alpha particle's orbit is 34.4cm (as the proton,) what magnetic field is required to steer the alpha particle down the 30° beamline?

$$B = \frac{m_{\alpha} v_{\alpha}}{q_{\alpha} R} = \frac{6.68 \times 10^{-27} \text{ kg} \times 1.256 \times 10^7 \frac{\text{m}}{\text{s}}}{2 \times 1.6 \times 10^{-19} \text{ C} \times 0.644 \text{ m}} = 0.7622 \text{ T}$$

3. What are the K_{α} and K_{β} wavelengths for the following elements: He, Li, Cu, Ni, Fe, Au? What are the corresponding x-ray energies? (**I've included the elements for problems 4, 5, 6 and, 8 in the table below**)

$E_{(n=3)}$ (eV)	$E_{(n=2)}$ (eV)	$E_{(n=1)}$ (eV)	Element	Z	hc (eVs)	ΔE_{α} (eV)	$\lambda_{\alpha}(\text{m}) = hc/\Delta E$	ΔE_{β} (eV)	$\lambda_{\beta}(\text{m}) = hc/\Delta E$
-1.511	-3.4	-13.6	He	2	1.24E-06	10.2	1.21875E-07	12.08889	1.02832E-07
-6.044	-13.6	-54.4	Li	3	1.24E-06	40.8	3.04688E-08	48.35556	2.5708E-08
-37.77	-85	-340	C	6	1.24E-06	255	4.875E-09	302.2222	4.11328E-09
-545.5	-1227.4	-4909.6	Ca	20	1.24E-06	3682.2	3.37604E-10	4364.089	2.84853E-10
-1184.7	-2665.6	-10662.4	Cu	29	1.24E-06	7996.8	1.55453E-10	9477.689	1.31163E-10
-1101.6	-2478.6	-9914.4	Ni	28	1.24E-06	7435.8	1.67181E-10	8812.8	1.41059E-10
-944.44	-2125	-8500	Fe	26	1.24E-06	6375	1.95E-10	7555.556	1.64531E-10
-8052.7	18118.6	-72474.4	W	74	1.24E-06	54355.8	2.28701E-11	64421.69	1.92967E-11
12513.5	28155.4	112621.6	U	92	1.24E-06	84466.2	1.47174E-11	100108.1	1.24178E-11
-9193.6	20685.6	-82742.4	Au	79	1.24E-06	62056.8	2.00321E-11	73548.8	1.6902E-11
-2540.2	-5715.4	-22861.6	Mo	42	1.24E-06	17146.2	7.25015E-11	20321.42	6.11731E-11
-8959.4	20158.6	-80634.4	Pt	78	1.24E-06	60475.8	2.05557E-11	71675.02	1.73439E-11

4. What is the ratio of the K_α wavelengths of uranium to carbon?

$$\text{Based on the table above: ratio} = 1.47 \times 10^{-11} \text{m} / 4.88 \times 10^{-9} \text{m} = 3.0 \times 10^{-3}$$

5. What is the ratio of the K_β wavelengths of tungsten to calcium?

$$\text{Based on the table above: ratio} = 1.93 \times 10^{-11} \text{m} / 2.85 \times 10^{-10} \text{m} = 6.8 \times 10^{-2}$$

6. If a spectrograph (a graph of the energy or wavelength spectrum) had a wavelength resolution of $\Delta\lambda = 10^{-12}$ m, would it be able to separate the K_α lines for platinum and gold? (Resolution means that anything smaller than this value, I won't be able to distinguish.)

$$\Delta\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \text{ Js} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \times 3 \times 10^8 \frac{\text{m}}{\text{s}}}{62056.8 \text{ eV} - 60475.8 \text{ eV}} = 7.86 \times 10^{-10} \text{ m}.$$

Since this is larger than 10^{-12} m I'll be able to see them as separate

7. What corresponding energy resolution for $\Delta\lambda$ given in problem 6?

$$\Delta E = \frac{hc}{\Delta\lambda} = \frac{6.63 \times 10^{-34} \text{ Js} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \times 3 \times 10^8 \frac{\text{m}}{\text{s}}}{1 \times 10^{-12} \text{ m}} = 1.24 \text{ MeV}$$

8. What are the two shortest wavelengths for a molybdenum atom?

The two shortest wavelengths are given in the table above and are 7.25×10^{-11} m and 6.12×10^{-11} m.

9. An unknown single element target is used in a PIXE experiment and characteristic x-rays are produced with wavelengths of 1.55×10^{-10} m and 1.31×10^{-10} m. What is the elemental make up of the target?

From the table above, it appears that the element is copper.

10. Show that the Moseley's law for K_α radiation may be expressed as

$$\sqrt{f} = \sqrt{\frac{3}{4} \left(\frac{13.6 \text{ eV}}{h} \right)} (Z - 1) \text{ where } f \text{ is the x-ray frequency.}$$

$$E_n = -\frac{13.6eV(Z-1)^2}{n^2} = hf \rightarrow f = -\frac{13.6eV(Z-1)^2}{hn^2} \rightarrow$$

$$f = \frac{13.6eV(Z-1)^2}{h} \times \left(\frac{-1}{2^2} - \left(\frac{-1}{1} \right) \right) = \frac{3}{4} \times \frac{13.6eV(Z-1)^2}{h}$$

$$\sqrt{f} = \sqrt{\frac{3}{4} \times \frac{13.6eV}{h}} (Z-1)$$